with any solid substance turn to crystals of ice and frost. Even larger raindrops often descend through the cold air and cover the limbs of plants and the ground with a layer of smooth ice, known as Glatteis to German meteorologists. We presume that this was the phenomenon recorded by Mr. Emigh.

If, however, he observed liquid precipitation amounting to 0.03 inches of water in forty-eight hours, and observed this to retain the form of water when measured in the rain gage, although the air temperature and that of the rain gage varied between 24° F. and 13° F., then this certainly is unprecedented.—*C. A.*

STUDIES ON THE DIURNAL PERIODS IN THE LOWER STRATA OF THE ATMOSPHERE.

By Prof. FRANK HAGAR BIGELOW.

I. THE DIURNAL PERIODS OF THE TEMPERATURE.

GENERAL REMARKS.

The following series of papers contains the results of a research into the periodic diurnal processes that take place in the strata of the atmosphere within two miles of the sea-level surface, as disclosed by the data derived from the balloon and kite ascensions made during the past ten years. It includes a discussion of the variations of the temperature, the pressure, the vapor tension, the atmospheric electric potential and coefficient of dissipation of the electric charge, and the diurnal periodic action of the magnetic force. These subjects have been under discussion by meteorologists for many years, but the issue has been so indecisive as to imply that certain important terms have been lacking in the problems, so that it was impossible to come to any definite view regarding the causes and effects in the physical processes. That all these diurnal periods depend upon the effects of the solar radiation in the earth's atmosphere has been evident, but the difficulty of matching together the various lines of experimental evidence derived from observations has been so great that no settled solution has seemed available. The additional data which have been recently secured through observations made in the free air above the ground have, however, altered the point of view in some respects, so that it is believed that the account to be given in these papers describes natural conditions more nearly than has heretofore been possible.

The immediate occasion for undertaking this research consists in the necessity of deciding upon the best lines of work for the Mount Weather Meteorological Observatory, at Bluemont, Va. The organization of so large an institution, dealing with problems in common meteorology, solar radiation, atmospheric electricity and magnetism, made it very important to acquire a clear idea of the relative values of the several types of observation, in order that suitable instruments might be installed and proper observations inaugurated. Since the effects of solar radiation involve many local characteristics which ought to be eliminated before the pure solar terms can be obtained, it was evident that some further knowledge of the diurnal verifications of the several elements should be secured if possible, at least to the extent of reconciling the conflicting evidence that the special lines of research have hitherto produced. It seemed the simplest course to make a study of the data furnished by kite and balloon ascensions, and for this purpose the observations at Berlin,[1] Trappes,[2] Hald,[3] and Blue Hill[4] have been studied.

In this paper our examples will be taken from the Blue Hill data as more applicable to the American meteorological field than the European data can be without special consideration. It should be noted that the Blue Hill Observatory furnished the Weather Bureau with certain temperature observations, made at the Valley Station, which were required in the proposed discussion, and for this courtesy our thanks are expressed.

[1] Wissenschaftlice Luftfahrten, 1888–1898, Berlin.

[2] Veröffentlichungen der Internationalen Kommission für wissenschaftliche Luftschiffahrt, 1901–3

[3] Travaux de la Station Franco-Scandinave de Sondages Aériens à Hald, 1902–3, L. T. de Bort.

[4] Observations at the Blue Hill Observatory, 1901–2, and appendix of the observations with kites 1897–1902, with discussion by H. Helm Clayton.

METHOD OF REDUCING THE OBSERVATIONS.

In Volume XLIII, Part III, Annual Harvard College Observatory, Table III, pages 166–214, the data are given for the temperatures on Blue Hill summit, 195 meters, at various heights, and occasionally at the Valley Station, 15 meters, together with the hour and minute of the observation.

(1) The first step in this discussion was to concentrate this material into smaller proportions by taking the mean values where the kites soared at about the same elevation. This gave a new series of data for the time, height, temperature at that height, and temperature at the summit. Corresponding temperatures for the valley at these times were extracted from the observatory records, at the request of the Weather Bureau, so that it became possible to refer the temperature-falls practically to the sea level. It was feared that any characteristic effects of the Blue Hill itself upon the diurnal temperatures, by means of radiation or by convection currents, might prevent the computed temperatures at higher elevations from bringing out the law in the free air with sufficient purity.

(2) A computation of the temperature-fall was next made for each time of observation by taking the difference between the temperature at the height and the valley temperature. A discussion of these temperature differences was preferred, in order finally to obtain the mean temperature at certain selected levels for each hour in the day, rather than to mass together the actual temperature readings recorded at these levels. In the former case the numerical values are less scattering than in the latter, and therefore they are more easily reduced to mean values. If the actual temperatures of the air, in the successive masses associated with the progress of high or low areas over a given station, are employed as the basis of computation, a very large number of observations are required to produce correct normal values in the several strata. The mean temperature falls, on the other hand, added to the normal values at the Valley Station, give the same result theoretically, and this can be obtained much more exactly for a limited number of observations by the method of differences.

(3) The first collection of the temperature differences contained the data applicable by simple interpolation to the levels 15, 195, 400, 600, ... 3800, 4000 meters, or as high as the ascension made its record. The data from the several years, 1897-1902, were collected by months, so that for example all of the January temperature-falls were brought together. They were also arranged by cyclonic and anticyclonic areas, so as to distinguish between the cold southward-directed current and the warm northward-directed current. The former covers generally the areas lying between the centers of the high and low to the eastward of the high, with winds from the northern quadrants, and the latter includes the areas between the centers of the low and high to the eastward of the low, with winds from the southern quadrants. Referring to the subareas adopted in my Cloud Report, chart 9, page 139, they were arranged in the following scheme, marked for convenience H. I=N.W., L. II=N.E., for southward, L. III=S.W., H. IV=S.E., for northward. The subarea for Blue Hill on the date of observation was scaled from the Weather Bureau daily weather maps.

The purpose of this collection was to discover to what extent the diurnal temperature-falls and corresponding gradients in the free air depend upon the cyclonic circulation, that is whether the temperature-fall is different over the cold currents from the north to that over the warm currents from the south. It may be stated in anticipation of the result that no important difference could be determined from these observations.

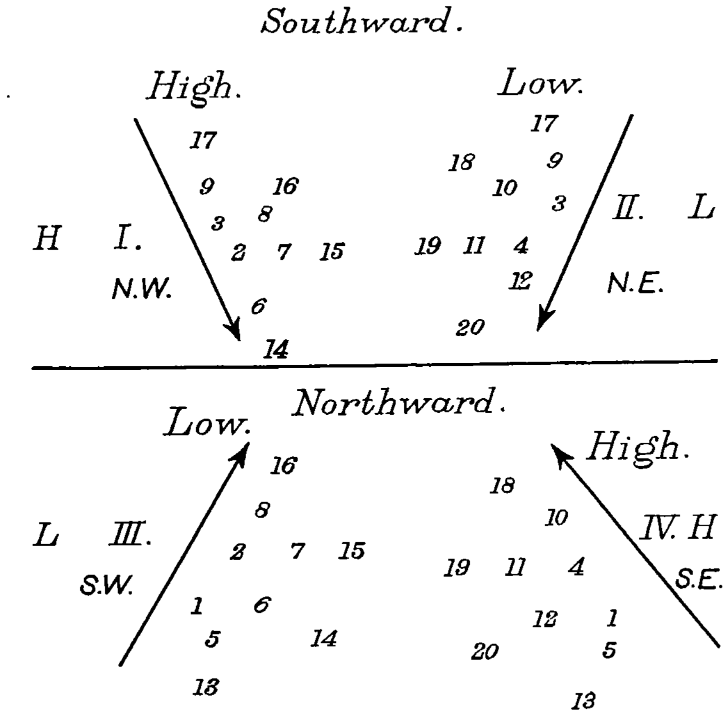

FIG. 1.—Adopted subareas for collecting the temperature data.

(4) In the next collection of the temperature-falls the northward data was kept separate from the southward data, both being computed independently. A further concentration was effected by interpolating the data to the values occurring at the hours 12 m., 1, 2........12 p. m., 1,........ 12 m., for each ascension, and bringing together all the data occurring at the same hour of the day, and as before for each month in the year at the adopted 200-meter intervals. These tables give the respective differences of temperature at each hour of the day, at the heights adopted, in warm and cold currents, and for each month in the year. From this point onward it was necessary to resort to a graphic construction to obtain average values, because the data were not sufficiently abundant at all hours of the day, in each month, for so many levels, to make the result reliable. There may be some question as to the definitive values of the results on this account, but as it was our purpose to obtain a provisional idea of the atmospheric conditions, for the purpose of planning further observations at Mount Weather, we were justified in making as much as possible out of the data in hand.

(5) The individual values of the temperature-fall for each elevation adopted were plotted on sheets, keeping each month separate, so arranged that the ordinates are the differences of temperature between that observed at the Valley Station and the temperature recorded by the thermograph attatched to the kite, the abscissas being the hours of the day beginning at midnight. The data belonging to the southward-directed, or cold currents, were plotted in black and the northward, or warm currents, in red, so that by inspection any systematic difference between these two types could be seen. It may be stated that *no notable divergence between the temperature-falls over the warm and cold currents could be detected*, and we conclude that the warm and cold masses resting on the ground, on either side of the cyclone center, fall off by about the same gradients up to at least one mile, and probably to two miles, in elevation. These masses of air, therefore, preserve their relative independence in the lower strata of the atmosphere, whatever changes may occur in their mechanical structure when penetrating the eastward drift of the higher levels. At the Valley Station itself the actual temperatures were plotted in the same way that temperature-falls were plotted for the several levels lying above it. Mean lines were now drawn through these several groups, or points, to represent as nearly as possible the average values. There was little difficulty in doing this accurately for the hours 10 a. m. to 8 p. m., where data were abundant, but for the remaining hours of the day there was more uncertainty, especially in the winter months, when the number of ascensions was not great. In the levels 195, 400, 600, 800 and 1000 meters, it was possible to construct fairly reliable mean curves, but since the scattering increases rapidly above the 1000-meter level, it was not possible to proceed directly by this method above the 1400-meter level.

(6) In order further to control these average curves and insure an accuracy even greater than could be obtained by simple estimation of the mean position of the curves, the following process was employed. For the hours 12 a. m., 4 a. m., 8 a. m., 12 p. m. 4 p. m., and 8 p. m., the values lying on these curves for the respective hours were transposed to another set of sheets, where the ordinates are the temperature-falls, and the abscissas the month in the year. Thus a rough locus was formed for 12 a. m. or midnight, so that the varying value at that hour for each month in the year might be seen. A curve was then drawn through these points, smoothing down the irregularities, on the theory that the temperature variation constructs a comparatively regular curve in the course of the year. This procedure doubtless tended to fix six points on the diurnal curve at each level with considerable accuracy, and these adjusted points were now transferred back into the preceding set of curves originally derived from the individual observations. The passage from month to month in the way indicated, allowed us to bridge over many gaps in the kite record that could not otherwise have been done. The first set of curves was now readjusted in conformity with these guiding points at 4-hour intervals. The adjustment of fragmentary records by continuous curves crossing each other in two directions as in this case, nearly at right angles, is not only expeditious but usually brings out results very close to the truth, because the mutual adjustment of neighboring points in two directions eliminates the merely accidental errors due to short and imperfect records. This is especially true if some of the fixed points are well established as in these observations during the afternoon hours of the warm months of the year.

(7) For the prosecution of this discussion whose object it was to eliminate the special local conditions pertaining to the Blue Hill summit, it was necessary to secure accurate normal values of the temperature at each hour of the day, and for each month of the year, at the Valley Station. As these data were not in hand at the Blue Hill Observatory we proceeded as follows:

As stated above the normal diurnal curves for the Valley Station were computed from the data supplied to the Weather Bureau at the times the kites were flying. Also, the normal diurnal temperatures for the summit were supplied from the records of twenty years 1885-1904, which were accurate. By the computations above described for the temperature differences between the summit and valley, inasmuch as the amount of data was sufficient, we had reliable corrections which applied to the summit normal temperatures for each hour gave the corresponding values at the Valley Station. These were compared with the diurnal values obtained on the days of kite ascensions, and from their combination certain surface temperatures were found upon which the temperatures computed in higher strata were made to depend. Any inaccuracies pertaining to the final results derived in this manner can be eliminated by further direct observations, but it will require a very large number of additional ascensions to accomplish any such purpose.

(8) We can impose upon this data yet another mutual adjustment. Up to this point the several curves for each month have been kept entirely independent of one another up to 1400 meters, but they were now brought together on sheets, one for each month, by transferring the several curves to the same set of axes of ordinates and abscissas. The successive curves in elevation now took positions appertaining to their respective temperatures differences. This can be seen by inspecting the curves of figs. 2–13, "Temperature-falls in the the lower strata, Blue Hill kite observations 1897-1902." In the lower levels 195, 400, 600, 800 and 1000 meters, there is great divergence in the shape of the curves, but they gradually approach a typical form which must be eventually that of the temperature curve at the surface itself. This is evident from the fact that at some elevation the diurnal variation proper of the temperature ceases to be effective, and since the temperature-falls were measured from the surface curve, the same curve must appear at those levels which have no true diurnal variation of their own. The difference between the surface curve and the computed curve at any given elevation gives the variation belonging to that level. The elevation at which the diurnal variation really disappears for each month in the year was not known, and could not be determined from the observations. It must be lower in winter than in summer, and I have merely assumed an average of 3400 meters. In case this is not correct, it yet is evident from the formation of these curves up to 1400 or 1600 meters that it has become a comparatively small quantity and that a change in the elevation from 3400 meters will have little effect upon the conclusions which we required in this series of papers, since they pertain to the strata up to only 2000 meters.

(9) The kite ascensions in several cases extended up to 3000 or even to 4000 meters, and by studying the computed table of temperature-falls it was not difficult to select the temperature-fall applicable at the 3400-meter level for each month. These were plotted in an annual curve, and the smoothed values were adopted for further use. At the value determined in this way for 12 m., or midday, as the temperature-fall for the month, the mean diurnal temperature curve of the Valley Station was plotted, and it is seen as the uppermost curve of the system marked 3400. We had already carried the other set of curves up to 1400 meters, and it was proper to suppose that the gradient system changed by regular steps between these two elevations. A study of these curves from month to month will, I believe, lead to the conviction that they are a very close approximation to the mean temperature-fall system in the lower strata which would be derived from a very long series of ascensions.

RESULTS OF THE DISCUSSION.

The system of curves, figs. 14–25, "Temperature-falls in the lower strata, Blue Hill kite observations, 1897–1902," contain the final results of this discussion. The chief point of criticism, as already mentioned, is the adopted height, 3400 meters, at which the surface curve should be located. The interpolation between this curve and the 1400-meter curve will be a little different if the topmost curve should preferably be placed at an elevation lower or higher than the one adopted, which is about two miles above the summit of Blue Hill. The most conspicuous feature of these curves in each month is the relative forms of the curves at 195, 400, 600, and 800 meters, which indicate that the temperature-falls are very different in the successive lower levels. By taking the differences between any two curves of the system the mean temperature gradient can be readily computed. Another striking characteristic is the persistent inversion of temperatures in the hours from 10 p. m. to 5 a. m., especially in the lower levels up to 1000 meters. This is seen by the ordinates being drawn with a positive sign, or downward on this scale of ordinates. The midday maximum temperature-fall can be seen to occur at an earlier hour in the lower levels, as 12 m. to 1 p. m., and at a later hour in the higher levels, as 2 to 4 p. m. The maximum rate of variation is quite uniformly located in the morning hours at 6 to 10 a. m. for rising temperature and at 5 to 9 p. m. for falling temperature. An examination of the curves from month to month shows that there is a general increase in the amplitude from winter to summer. At the same time, in the winter months the amplitude for the lower levels, 600 to 1000 meters, is greater than for the upper levels; on the other hand, in summer the amplitude in the lower levels is less than in the higher levels. The transition months, April, May and September, October, have about equal amplitudes in each level. The fact that this subtle law has been deduced by the method of computation employed speaks strongly for the efficiency of cross-plotted adjustments.

I have used the same method in deducing the temperatures of the atmosphere up to 16,000 meters, charts 78, 79, International Cloud Report, and in determining the temperatures under the Rocky Mountain Plateau at sea level, chart 13, Barometry Report, and in other places. It is the only satisfactory way to adjust broken series of incomplete data to an approximate mean, such as can be secured otherwise only by a very great number of direct observations. Several other important relations will be found in the other papers of this series which tend to confirm these conclusions. It may also be further noted that there seems to be a semiannual period in the positive or the inversion ordinates in the morning hours 2 to 5 a. m., as well as a rearrangement in the order of heights at which this is greatest. Thus, the amplitude for the 4 a. m. hour of the 400-meter curve has a single period, with maximum in February and minimum in October; the 600-meter curve, however, has a maximum in February and another maximum in June, with minima in April and October; the 800-meter curve has two maxima and two minima in agreement with the 600-meter curve. The 195-meter curve at the summit of Blue Hill shows that the temperatures at the summit and base, 15 meters, do not vary in parallel, and hence the gradient system referred to the level of the open country will differ somewhat from that referred to the summit. This should be remembered in the use of the gradients of the Blue Hill Observatory Report.

We have now obtained the material necessary for deducing the mean (approximate) temperatures at the different levels, by merely adding algebraically the temperature-falls to the normal temperatures of the Valley Station. The results are given in the tables, figs. 14–25, "Blue Hill temperatures in the lower strata." The ordinates of temperature have been plotted to decrease upward, in order that they may conform to the actual conditions in the free air, where the temperature diminishes generally with the height. There are several results of unusual interest to meteorology which appear on the face of these charts. The first is the remarkable distribution of temperature in the levels from 600 meters upward as compared with the surface temperatures. In the winter months—December to March, inclusive—there is a pronounced inversion of temperature throughout the day, so that the night hours, 7 p. m. to 5 a. m., are warmer, while the day hours, 6 a. m. to 6 p. m., are colder than the mean for the day at the several levels. It seems very remarkable that in the hours of full sunshine the effect of the radiation on the temperature of a stratum of air should be to allow it to remain cool rather than to heat it. Evidently the result comes about indirectly, by reason of the fact that the incoming short-wave radiation has little influence directly on the temperature, because there is not much absorption. These short waves impinge upon the surface of the earth, which becomes a radiating body of low temperature and emits long waves. These are strongly absorbed according to the prevailing physical conditions, as the relative amounts of dry air and aqueous vapor, coefficients of absorption for different wave lengths, and so on. Convection currents also enter into the result, and, indeed, the complex function here displayed requires much careful examination before further conclusions can be stated. The temperatures diminish generally with the height after leaving the 400-meter level, but the diurnal period gives a maximum of cold at midday and a minimum about midnight. In the summer months, on the other hand—June, July, August—the inverted temperature distribution does not exist relatively to the surface, but the curves are all of the same general type, with a maximum temperature in the late afternoon, and minimum in the early morning. In the transition months—April, May, September, October, November—the diurnal temperature curve has two maxima, 8 a. m. and 8 p. m., and two minima, 2 a. m. and 2 p. m. The process of transition can be followed in the several levels from month to month, and it is a very interesting phenomenon.

The second important feature of these curves is the building of a semidiurnal period in the temperature at the elevation 400 to 600 meters, in all months in the year, with the maxima at 8 to 9 a. m. and 9 to 10 p. m. They are seen very distinctly represented in May and September, where they are formed up to the very top of the diurnal disturbance. The single diurnal period at the surface is replaced by a double diurnal wave at 400 meters, and this appears quite plainly in every month except July, where it probably is nearly extinct. In the higher levels, above 800 meters, there is a tendency for the double periods to contract the maxima from the 9 a. m., 9 p. m. hours nearer toward midday, and form two crests or a single crest near midday, especially in the winter months. It will be shown in the next paper of this series that those superposed temperature waves, having their maxima disposed as just explained, are competent to produce the diurnal variation of the barometric pressure in the single, double, and triple components, into which the observed pressure at the surface is usually resolved by the Fourier Series of Harmonics. Mr. Clayton has obtained similar curves of temperature at 500, 1000, 1500 meters, as shown on fig. 5 of his paper on "The diurnal and annual periods of temperature."—Annual Harvard College Observatory, Vol. LVIII, Part I, 1904, though they are composites of the several curves really belonging to different months of the year. It will be seen from our curves that mean annual values computed from observations taken in all parts of the year are correct only for certain limited intervals, in which the varying temperatures pass through such special values. Similarly, all discussion of data depending upon mean values made up in this way can have only a limited application in deducing daily free air temperatures throughout the year. This disclosure of the fact that the temperature curves differ according to the elevation from the one observed at the surface opens up the possibility of explaning not only the semidiurnal and triple diurnal barometric waves, but also the movements of the ions in the atmosphere in their relation to the electric potential gradient, the coefficient of neutralization and number of ions, in the connection with other meteorological phenomena, and the variations of the diurnal magnetic field in all latitudes of the earth. These researches will be explained in the other papers of the series.

MATHEMATICAL THEORY OF ICE FORMATION.

By S. TETSU TAMURA.

It is well known to the mathematical physicist that the first to give the most complete mathematical theory of heat conduction was Joseph Fourier, who belonged to the constellation of the most brilliant men of science in the time of Napoleon Bonaparte. The entire subject of heat conduction was created by Fourier's "Théorie Analytique de la Chaleur", and all its later experimental developments have been suggested, or rendered possible, by this immortal work. Side by side with La Place's and Poisson's Equations, Green's Theorem, and Lagrange's Equation, Fourier's Theory, or the analysis known to mathematicians as Fourier's Method, has played a wonderful rôle in the whole domain of mathematical and experimental physics. Lord Kelvin once said that Fourier's work is one of the greatest contributions to the nineteenth century. "Fourier's exquisitely original methods," said the late Professor Tait, "have been the source of inspiration of some of the greatest mathematicians, and the mere application of one of its simplest portions to the conduction of electricity has made the name of Ohm famous".

Among those problems which afford us examples of the application of Fourier's beautiful analysis, there are two groups of the geophysical questions which are extremely important, as well as absorbingly interesting. To the first group belong the problem of the distribution of the earth's internal heat and its consequent effect upon the temperature near the earth's surface, and the problem of the penetration of the sun's heat into the crust of the earth. These problems of widely general and of peculiar mathematical interest were first attacked by Fourier himself and Poisson, the greatest contemporary of Lagrange, and La Place. Some forty years ago, Lord Kelvin startled geologists by telling them that Fourier's Theory forbids such long intervals of time as they were in the habit of assigning to the aggregate of paleontological phenomena. The problems of the second group are rather of smaller feature, but none the less beautiful and interesting as the applications of Fourier's analysis. They are problems of ice formation, especially in the polar sea, and of the penetration of frosts into the ground. These questions, with whose discussion we are now concerned, seem to have been attacked by very few scientists, notably by Franz Neumann who was an eminent mathematical physicist of his day, and Julius Stefan, the famous physicist. Neumann's analysis[1] appears in Riemann-Weber's Partielle Differentialgleichungen der Mathematischen Physik, Bd. II, § 49, and Stefan's memoir[2], "Ueber die Theorie der Eisbildung, insbesondere über die Eisbildung in Polarmeere" in Wied. Ann. Bd. XLII.

Suppose that the surface of a mass of water is in contact with another surface whose temperature is θ_s. θ_s may be either constant or variable; but it must be always below freezing. Under this surface there will be formed a layer of ice, whose thickness, ε, is a function of the time, t. The temperature, θ, of the mass of ice is itself a function of the time, t, and of the distance, x, from the surface.

Let

$$\left.\begin{aligned}
\theta_1 &= \text{temperature in ice.}\\
\theta_2 &= \text{temperature in water.}\\
k_1 &= \text{conductivity of ice.}\\
k_2 &= \text{conductivity of water.}\\
\rho_1 &= \text{density of ice.}\\
\rho_2 &= \text{density of water.}\\
c_1 &= \text{specific heat of ice.}\\
c_2 &= \text{specific heat of water.}\\
a_1^2 &= \frac{k_1}{c_1\rho_1} = \text{diffusivity of ice.}\\
a_2^2 &= \frac{k_2}{c_2\rho_2} = \text{diffusivity of water.}
\end{aligned}\right\} \text{Assumed as constant.}$$

Then the following equations hold:

$$\begin{aligned}
\frac{\partial\theta_1}{\partial t} &= a_1^2\frac{\partial^2\theta_1}{\partial x^2} \text{ in ice, or for } 0 < x < \varepsilon.\\
\frac{\partial\theta_2}{\partial t} &= a_2^2\frac{\partial^2\theta_2}{\partial x^2} \text{ in water, or for } \varepsilon < x.
\end{aligned} \qquad (1)$$

[1] Also in Wien. Ahad. Sitz. ber. Bd. 98, Abth, II a.
[2] The title, "Vordringen des Frosts."